A Possible Proof of Goldbach's Conjecture

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Statement of Conjecture

Goldbach's Conjecture, which was announced in 1742, asserts that each even positive integer greater than or equal to 4 is the sum of two prime integers. Thus, e.g., 12 = 5 + 7. The Conjecture is still unproved.

The following Possible Proof — presented in two versions — is based on a strategy that has been effective with two other very difficult problems that we believe we have solved. The strategy consists in finding a structure that contains all possibilities and that shows crucial relationships between them.

In the present case, that structure is the set of all "diagonals". There is one diagonal for each even positive integer $2k$. The diagonal contains all pairs of odd positive integers that sum to $2k$. If a counterexample to Goldbach's Conjecture exists, it will appear in a diagonal.

Preliminaries

1. *Definition*: the *matrix* N is the infinite matrix whose rows and columns are labeled 3, 5, 7, 9, ... through all the odd positive integers.

Each cell of the matrix has coordinates (row, column). The contents of each cell is the sum row + column.

(For ease of understanding the following, the reader is encouraged to draw, and fill in, the matrix N for rows and columns 3, 5, 7, 9, 11, 13.)

2. *Definition*: The matrix N contains an infinite sequence of *diagonal*s. For each $n \geq 1$, the nth diagonal is associated with the even positive integer $2k = 2n + 4$. There are n cells in the nth diagonal. Each cell in the diagonal has, for its coordinates, a pair of odd positive integers that sum to $2k$. Thus each cell contains the same even positive integer, namely, $2k$.

We call each cell a "pair".

(The term "diagonal" is derived from the fact that all the elements of each diagonal lie on a diagonal line that is perpendicular to the main diagonal of the matrix N.)

3. For each $n \geq 1$, the nth diagonal is constructed as follows:.

We begin with the downward sequence $s(n)$ of left-hand elements in all pairs. That downward sequence is simply the sequence of n successive odd positive integers, starting with 3.

The first element is 3. There is no possibility it can be something else.

The next element is the next odd positive integer, namely 5. There is no possibility it can be something else.

The next element is the next odd positive integer, namely, 7. There is no possibility it can be something else.

…

[We proceed in this manner, through the successive odd positive integers up to and including the nth.]

The downward sequence $r(n)$ of right-hand elements in all pairs is just the reverse of $s(n)$, so there is no possibility that any of these can be something else.

.

(1)

Therefore there is one and only one possibility for each diagonal.

By way of an example, the following is the sixth diagonal. It is associated with $2k = 2(6) + 4 = 16$.

(3, 13)
(5, 11)
(7, 9)
(9, 7)
(11, 5)
(13, 3)

4. *Definitions*: A diagonal having no coordinates (p, q), where p, q are primes, we call a *counterexample diagonal*, because $2k$ is then an even positive integer that is not the sum of two primes, contradicting Goldbach's Conjecture.

A diagonal having at least one pair of coordinates (p, q), where p, q are primes, we call a *non-counterexample diagonal.*

Possible Proof (First Version)

1.Statement (1) implies that there is one and only one set of diagonals, regardless if counterexamples exist or not.

2. But that implies that if the nth diagonal is a counterexample diagonal, it is also a non-counterexample diagonal, which is absurd.

Therefore counterexample diagonals do not exist, and Goldbach's Conjecture is true.

Possible Proof (Second Version)

1. If one or more persons do not know if a given even positive integer $2k$ is associated with a counterexample diagonal or a non-counterexample diagonal, then for those persons there are two possibilities: (1) the diagonal is a counterexample diagonal, or (2) the diagonal is a non-counterexample diagonal.

(*Note*: The Goldbach Conjecture research community knows, by computer test, that at least for all j, $1 \leq j \leq m$, the jth even positive integer $2k$ is associated with a non-counterexample diagonal. The community knows that $m > 10^{18}$.)

2. Since, at the time of this writing, the Conjecture has not been proved or disproved, and given the *Note* in step 1, and the fact that there is an infinity of even positive integers 2*k* greater than *m,* there exists an infinity of 2*k* for which there are the two possibilities specified in step 1.

3. Steps 1 and 2 imply that there are two possibilities for an infinity of even positive integers 2*k*, the smallest of these being greater than *m* in the *Note* in step 1.

4. However, in "Preliminaries" on page 1, we showed that

(1) "*... there is one and only one possibility for each diagonal".*

Thus we have a contradiction that is brought about by the possibility that a diagonal can be a counterexample diagonal, and therefore Goldbach's Conjecture is true. (The contradiction is not brought about by the possibility that a diagonal can be a non-counterexample diagonal, because we know, by computer test, that it can.)