

Index

Index

Symbols

=
substitutes for 109

A

academic math teaching
as relic of the Middle Ages 13, 121
academic papers
as miniature Environments 39
Advanced vs. Intermediate courses 14
AI -- see "artificial intelligence"
"AI Spring" 131
"AI Winter" 131
algorithm 55
"all-or-nothing" approach to mathematics 39
"All there is is knowledge."(in the academic world) 23
alphabetical order 61
as Fundamental Concept 61
applied mathematics
scandal of modern pure mathematicians' contempt for 177, 181
approximation
in problem solving 52
of a subject by logical vs. problem-solving structure -- graphical image analogy 68
artificial intelligence
and Environments 131
use of in proving theorems 154
Asimov, Isaac
on the periodic table of the elements 167
astronaut in space suit as analogy to person using Environment 51

B

basics
going to school to learn 122
beg the question 126, 137
Bergson, Henri
on cinematographic thinking 91

Index

Berkeley, Bishop
 on infinitesimals (fluxions) 156
"best schools"
 what do they offer that the others don't? 17
"Big Picture" 9, 39, 72, 149, 224, 227
Big Picture of the whole of mathematics 226
bird's eye view of the whole of mathematics 226
Bittinger, Marvin L., "Logic and Proof" 98
Bourbaki 228
boxes and arrows
 as description of most technical subjects 153
breaking complex things up into simpler things 47
briefing
 as a function performed by an Environment 22
 on space shuttle technology 22
building blocks
 of an entity 78
building, height of, student's solution to determining 42

C

calculus
 a reason why calculus is difficult 28
 "why can't I learn calculus in 15 minutes?" 21
 a briefing on the 21
 having to learn it for a special mission 21
 "A calculus textbook is a great big calculator." 143
 two of the best textbooks (in the author's opinion) 27, 32
 what, in Newton's physics researches, motivated him to investigate instantaneous
 rates of change? 157
 why "the" calculus? 157
"Calculus is the kind of thing that belongs in a machine." 143, 145
calculus lecture at University of California at Berkeley 13
category theory 122
Chaitin, Greg 187
China
 "a mandarin's education...was, mainly, learning to read" 141
 learning to read in ancient 141

Index

Chinese

why didn't Chinese develop easier-to-learn writing system? 24

chunking 49

cinematographic presentation 87

class of problems

more important than individual problems 149

program to solve a 36

classes of problems 9

addressed by an Environment 66

more important than individual problems 37, 147, 149

classifying as thinking 152

classifying problems

beginning a course by 149

classroom course

used as problem-solving device 69

classroom courses

as inefficient ways of learning 123

classroom teaching

student behavior model implied by 93

cleverness as requirement of good programming 140

"The Club"

a way of limiting access to 100, 104

definition of 100

complete Environment 18, 84, 117, 122, 124

definition of 69

complexity of mathematical concepts (entities) revealed by Environments 83

computer

use of in calculus 143

computer graphics -- see also "graphics"

computer industry

as type of Hell 121

computer science

has healthier attitude than mathematics toward notation

computer scientist who worked all problems in textbook 35

computer user

as "central processing unit" 50

"Contrary to popular opinion, mathematics is not a language..." 108

cookbook

Index

- Environment as a 127
- cookbook approach
 - Environment as a 145
- cookbook procedures
 - students desire for in order to solve problems 128
- Courant, R.
 - "Differential and Integral Calculus" 27
- course
 - classroom, used as problem-solving device 69
- course notes
 - used as a problem-solving device 69
- curl function 146

D

- data 70
- data base
 - business 69
 - Environment as a 67, 69, 70, 117
- data structure
 - abstract 77
- data type
 - list 77
- "decoding" of a proof 103
- definition
 - of terms, recursive 75
- derivatives
 - algorithm for finding 145
- Descartes, Rene
 - and analytic geometry 58
- Dijkstra, Edsger W. 49, 140
 - on importance of good notation 135
- Dirac, Paul
 - interest in arts and humanities 180
- directions -- see "procedure"
- divergence function 146
- "Do the job once" (of making notes, i.e., in an Environment) 93
- documentation of software 72

Index

- drawing
 - numbers to tell how to read a 91
- drawings
 - use of in an Environment 87
- Dyson, Freeman
 - against academy snobbery that looks down on inventors 182
 - Ernest Rutherford was largely self-educated 184
 - on Feynman's geometric approach to physics 146

E

- "Each student must learn for himself" 161
- EDM
 - definition of 71
- education PhD candidate 19
- Education PhDs 19
- education theory 19
- efficient Environment -- see under "Environment"
- Einstein, Albert
 - on his dislike of being forced to memorize for exams 94
 - on importance of pictures in understanding mathematical concepts 60
- electronics
 - as a mapping from functions to circuits 59
- "encoding" of a proof 103
- "Encyclopedic Dictionary of Mathematics" 71
- engineers
 - bad habit of wanting to design everything from scratch 46
- entity
 - definition of 73
- Environment
 - arguments against the idea 74
 - as a "cookbook" approach 145
 - as form of "pre-learning" 93
 - as means of obtaining a briefing on a subject 22
 - as nondeterministic "program" 50
 - as prosthesis 130
 - as source of self-confidence 124
 - classes of problems addressed by an 66

Index

- complete 18, 84, 117, 122, 124
- complete, definition of 69
- criticisms of the Environment concept, and replies thereto 127
- efficient, definition of 69
- efficient, structure of 72
- example of (partial) one for number theory 203
- how idea was developed 129
- means of implementing 72
- partial, ways of building a 84
- pencil-and-paper 126
- precursors of 70
- reveals the true complexity of mathematical concepts (entities) 83
- scales upward better than logical construction of subjects 120
- shortcuts to build a 74, 84
- student model implied by 93
- testing the worth of an 130
- what an Environment is not 117
- where to put examples 81
- why Environment concept is inevitable 118
- why the concept was developed 121
- "Build for when you have forgotten everything!" 93
- "Put the intelligence in the Environment!" 72
- Environments
 - and artificial intelligence 131
 - computer implementations of 138
 - make all math subjects look "the same" 121
 - pencil-and-paper 125, 126
 - types of 69
- equals sign
 - invention of 110
 - substitutes for 109
 - various meanings of 157
- Euclid
 - on learning 127
- "Every mathematical subject is a table" 30
- examples
 - where to put in an Environment 81
- exams

Index

guidelines for taking 170
exercises
 at end of traditional textbook chapters should all be referenced in Environment 79
explosion of mathematical knowledge 119, 122
"eye logic" 86

F

factors
 as building blocks 78
fear factor in mathematics 2
feedback
 importance of learning from 155
Feynman, Richard
 geometric approach to physics 146
 on learning biology 137
 on value of briefings 22
"force field" of intimidation surrounding mathematical subjects and concepts 2
format
 as partial substitute for prose 106
formulas
 students desire for, in order to solve problems 128
function concept 58
fundamental intuitions 114

G

Gauss, Carl Friedrich
 as student of classical languages 179
 on ideas vs. notation 136
gestalt 86
gestalt type of learning 29
global view
 Start Page gives a global view of a subject 72
global view of subjects 227
"Goedel, Escher, Bach: an Eternal Golden Braid", by Douglas R. Hofstadter 166
"going-around-ness" (repetition) 163
"Good enough is perfect" 53

Index

gradient function 146
graphics
 use of to show structure of subject 226
graphics to show structure of a math subject 224

H

Hamilton, William Rowan
 interest in languages 179
heuristic 55
heuristics
 for doing proofs 94
 inclusion of in an Environment 94
high school student and What vs. How 42
histories of mathematics
 best one I have ever come across 197
 most are dull recitations of who proved what when 197
Hofstadter, Douglas R.
 on "closeness" of mathematical ideas 166
Hollingdale, Stuart
 on Einstein's education 94
 on Gauss's study of classical languages 179
 on Hamilton's interest in languages 179
Holt, John 20, 144
 on education foundations 19
 on nature of intelligence 114, 171
"How do you know this?" 161
"How much should you be allowed to look up?" 137
"How to Create Zero-Search-Time Computer Documentation" 72
hypertext
 faster than page turning 125
 paper implementation of 71
 straight talk on 118

I

"I don't want to learn, I want to do!" 127
"I haven't time to enter all that information!" 74

Index

- ideas
 - importance of in mathematics 136
 - "If all it (a textbook) is is not wrong, then it's wrong!" 25
 - "If I don't understand the motivation, I don't understand the concept." 75
 - "If I were any good ..." 80
 - "If one learns, why not all?" 161
 - "If you want to tell the time, you need to know how the watch is made" 7, 64
- iff, meaning of 99
- ignorance
 - ways of concealing 170
- illustrations -- see "drawings"
- implement 45
- implementations
 - computer implementations of Environments 138
- index
 - absence of in textbooks 25
- index of a textbook as partial Environment 84
- indirect proof, meaning of 99
- infinitesimals
 - Bishop Berkeley's essay on 156
- Ingalls, Daniel
 - on information content of pictures 147
- input/output operations, examples of 58
- integrals
 - classification of 167
 - disgrace that development of heuristic for evaluating integrals is not part of every calculus course 37
 - heuristic for evaluating 36, 145
- integrated circuits
 - placement and routing of transistors in 131
- intellectual labor
 - Environment as a means of reducing 82
- intelligence
 - includes knowing what not to waste your time on 20
 - logical structure (Environments) as a means of putting intelligence, not merely knowledge, into a presentation of a subject 66
- intelligence tests 36
- interactive systems 154

Index

Intermediate vs. Advanced courses 14
intuition 114
IQ tests 36
"Is this important or it is just mathematics?" 98

J

journals, see "mathematics journals"
jr. high school student I tutored 170
"just-in-time inventory" 66
justification for statements in a proof 103
"just-in-time learning" 66, 120, 125, 146
just-in-time learning
 inevitability of as mathematical knowledge increases 17

K

Kaku, Michio
 on growth in scientific knowledge 119
Kellogg, Oliver Dimon
 "Foundations of Potential Theory" 27
Klein, Felix
 on following a geometrical argument 147
Kline, Morris
 "Why the Professor Can't Teach" 24
 on Felix Klein 147
 on importance of history of mathematics 196
Kline, Morris, "Calculus
 An Intuitive and Physical Approach" 27, 32
knowledge
 in the academic world, it is all that matters 23
 mathematical, explosion of 72, 119, 122
"Knowledge is not where it's at!" 23
Knuth, Donald 89
 "Mathematical Writing" 104, 105
 against his belief that a computer program is a piece of literature 105

L

Lamport, Leslie

papers on structured proof 104

learning

gestalt approach vs. look-up-able approach 29

not always your No. 1 task 22

"Learning is a red herring." 127

learning something familiar vs. learning something new 121

lemma, definition of 99

lemmas

look-up-ability of lemma statements 7

standard form for 108

lemmas and theorems you are allowed to use in a proof 80

LIFO, definition of 58

linear algebra

Environment for 72

linear search 126

list data type

ways of implementing in a computer 77

Littlewood, J. E.

on function concept 58

on importance of giving short, clear idea behind proof 112

on use of pictures in proofs 111

logical order 10, 126

logical structure 124

advantages of 65

definition of 64, 65

student behavior model implied by 93

logical structure vs. problem-solving structure

comparison via graphical image analogy 68

look up

how much should you be allowed to 137

look-up-ability

test of value of in problem solving 5

"look-up-able" 26, 51, 137

look-up-able

"Whatever can be made look-up-able, should be made look-up-able" ii

what should be made look-up-able 138

M

Mandelbrot, Benoit 187

manual, instruction
 organization of 6

"map"
 of a subject 144, 226
 of geometric figures 65

map
 use of to drive from Berkeley to Palo Alto 64

math
 reading it on a bus or subway 22

math teachers
 having them solve problems in front of the class 17

"math without words" 92

mathematical concept
 meaning of in this book 10

mathematical entity
 definition of 73

mathematical knowledge
 explosion of 72, 119, 122

mathematicians
 criticism of the often-inadequate indexes in their textbooks 5
 criticism of their assumption that mathematics is best taught in a classroom 13
 criticism of their assumption that the best way for a student to learn to solve mathematical problems is to sit in a classroom and, in effect, have the professor read the phone book to them 17
 criticism of their attitude toward teaching 18
 criticism of their low opinion of the computer 153, 176
 criticism of their practice of not including heuristics in published papers 140
 criticism of them as textbook writers 24
 greatest of all time 133
 myths about 175
 percentage that think in pictures 60

mathematics
 amount of 119

Index

- applied
 - scandal of modern pure mathematicians' contempt for 181
 - bird's eye view of the whole of 210
 - fear factor in 2
 - ideas, not notation, is fundamental 136
 - is not a language 108
- "Mathematics Dictionary" 70
- mathematics journals
 - mathematicians should be able to pay someone to edit their papers into the required style 107
- McLuhan, Marshall
 - books by 61
 - on source of idea of breaking complex things into simpler things 48
- memorization 82, 92
 - as byproduct of repeated use 93
 - compared to rapid look-up-ability 6
 - Environment as an aid to 93
 - in biology 137
 - why it is needed 123
- memorizing formulas and procedures 145
- Middle Ages
 - academic math teaching as relic of the 13, 121
- middle school -- same as "jr. high school"
- mimes
 - use of to teach math 91
- mistakes
 - learning from 156
- model
 - of typical student implied by logical structure vs. problem-solving structure 93
- moment of inertia
 - student's difficulty in understanding 158
- Moon Lander 52
- "Mother's Day Meditations from the Computer Room" 56
- motivation
 - value of understanding the motivation that led to discovery of a concept 75
- myths about mathematicians 175

Index

N

new

good ideas don't have to be 155

Nietzsche, Friedrich

on orators vs. thinkers 19

"not human alone, or machine alone, but human and machine acting as a unit" 50

"not problems but classes of problems" 37

notation 135

computer scientists have healthier attitude toward it than mathematicians 136

mathematical 135

structure is 136

number theory

example of (partial) Environment for 203

number theory, advanced

brief, informal overview of 224

O

orators vs. thinkers 19

outlining term papers 49

overachiever 52

overachievers

belief that studying self-improvement books is a sign of weakness 155

P

page

regarded as a picture 86

page turning in pencil-and-paper Environments 125

papers -- see "academic papers"

paradigm shift

alphabetical Environments as a 125

Parkinson, C. Northcote

on work and the time to do it in 21, 24

Parkinson's Law 21

partial Environment 117

definition of 69

partial Environments

Index

- ways of building 84
- pedantry
 - in style guides for writing mathematics 104
- pencil-and-paper Environments 72, 125, 126
- Perfect Student 16
- perfection
 - pursuit of 52
- periodic table of the elements 167
- phone book
 - traditional classroom course as having the phone book read to the students 17
- physics
 - correctly labelling a drawing representing the problem should be enough! 150
- physics problems
 - labelling the drawing should be enough! 150
 - use of spread sheet in solving 150
- picture
 - a page regarded as a 86
- pictures
 - as Fundamental Concept 60
 - difficulty of including in textbooks 60
 - information content of 147
 - reasoning from 60
- pictures -- see "drawings"
- placement and routing problem 131
- Polya, G. 140
- popularization
 - used as a problem-solving device 69
- practical applications of a mathematical subject
 - perfectly legitimate for students to ask what these are 182
- "pre-learning"
 - via using Question/Answer technique 164
- pre-learning
 - Environment entries as a form of 93
- prerequisites for a course 10
- principle entity
 - definition of 73
- print
 - against it as best means in all cases for conveying mathematical ideas 89

Index

- problem solving
 - as successive approximation 52
 - relation between speed and procedure for 38
- problems
 - classes of 9
 - importance of knowing where a problem "fits" relative to other problems 150
 - working all in a textbook 35
 - working all vs. writing down procedure 35
- problem-solving
 - by math teachers in front of class 17
 - why it is unimportant to primary, middle, and secondary school teachers 143
- problem-solving craft 141
- problem-solving productivity
 - suppose nation's victory in a war depended on 129
- problem-solving structure
 - definition of 66
 - student behavior model implied by 93
 - types of 69
- problem-solving structure vs. logical structure
 - graphical image analogy 68
- procedure
 - to drive from Berkeley to Palo Alto 64
- Professor Tax 18
 - paying the 18
- professor tax
 - paying the 21
- professors
 - can't have it both ways -- being paid to teach and doing nothing to improve their teaching 18
 - naivete of 129
 - what they want most 121
 - would rather do research than teach 152
- programming
 - relation between program speed and program correctness 38
- programs
 - operations on 76
 - similarity to proofs 99
- proof

Index

- against current word-centered, prose-centered, narrative approach to proof 106-107
 - “A proof is a story” 106
 - "decoding" of a 103
 - "encoding" of a 103
 - how a structured proof works 102
 - importance of giving justification for each assertion 25
 - justification for statements in a 103
 - level of knowledge assumed in a 103
 - maximum number of steps (seven) at a given level 102
 - structured, papers on 104
 - proofs
 - heuristics for doing 94
 - how to list techniques in an Environment 110
 - how to number steps in 109
 - how to write down, using an Environment 100
 - importance of giving short, clear idea behind proof 112
 - similarity to computer programs 99
 - use of pictures in 111
 - why difficult to understand in textbooks and lectures 100
 - properties
 - an entity, importance of listing 79
 - prose
 - format as a means of reducing need for 106
 - get rid of 136
 - prosthesis, Environment as a 130
 - publish-or-perish
 - was not always the rule for mathematicians 187
 - "Put the intelligence in the Environment!" 72
-
- ## **Q**
- quantum mechanics 73
 - Question/Answer technique for understanding difficult concepts 163
 - Question-Answerer 46
 - questions
 - respect your own 156
 - quotient concept 163

R

- rapid access to information in an Environment 146
- read
 - learning to read in ancient China 141
- reasoning modules 133, 135
- red herring
 - learning and understanding as a 127
 - learning as a 127
 - understanding as a 127
- reformer
 - don't become a 199
- "Render unto the machine, that which is the machine's, and unto humans, all the rest." 145, 143
- repetition ("going-around-ness") 163
- representing a mathematical entity
 - ways of 75
- roadmap -- see "map"
- rocket, DC-XA reusable 53
- Russell, Bertrand
 - "The History of Philosophy" 91
 - function of teacher is not primarily to dish out information 137

S

- Sagan, Carl
 - on his education at University of Chicago 181
- "same"
 - Environments make all math subjects look "the same" 121
- scalability 120
- scandals in the teaching of mathematics 142
- school
 - as place to learn basics 122
 - in the age of Environments 39
- Schorer, Peter
 - "How to Create Zero-Search-Time Computer Documentation" 72
 - "Shaving With Occam's Razor" 38
- searching as thinking 152

Index

- self-confidence
 - Environments help build 124
- self-improvement 155
- self-teaching 122
- semantics vs. syntax 45
 - related to the What vs. the How 45
- series
 - operations on 76
- seven, as maximum number of steps at any level in a proof 102
- "shape" of a math subject 224
- shortcuts to building Environments 84
- Socrates on memorization 92
- soft calculator
 - an Environment as a 66
- software design 50
- specialization 119
 - in mathematics 119, 181
- specialty
 - means of solving all problems in a specialty probably not found in speciality itself 119
- speed in problem-solving
 - proper place of 36
- speed in solving math problems 36, 145
- spread sheet 151, 152
 - as means of representing classes of physics problems 150
- "Start" Environment 139
- "Start" Page
 - contents of 72
- statistics program
 - graduate student's struggles to use 19
- Stein, Sherman, "Calculus and Analytic Geometry" 27, 32
- Stewart, Ian 119
 - on amount of mathematical knowledge 119
 - on mathematical symbolism 136
 - on mathematicians who think in pictures 60, 89
- structure
 - as notation 136
 - can do some of your thinking for you 136

Index

- does not change content of a subject 64
 - in computer programming 48
- structure conscious 52
- structured programming 49
- structured proof
 - how it works 102
 - papers on 104
- student's notes
 - used as a problem-solving device 69
- studentcraft 160
- students
 - how do exceptional ones hold their knowledge in their minds? 39
 - what they want most in a math course 121
- studying a mathematical subject "horizontally" vs. "vertically" 163
- summer vacation
 - returning to a subject after 121
- symbols --- see also "notation"
- synonyms 87
- syntax -- see also "notation"

T

- table
 - every mathematical subject is a 30
 - is a picture of a technical concept, or of a relationship between technical concepts 30
- tax laws 151
 - mathematics seen as no different than the 224
- teaching
 - function should not be merely dishing out information 137
 - why is math teaching as it is? 152
- technical subject
 - Environment works for any 69
- telephone book
 - typical math or technical course is essentially like having someone read the telephone book to you 127
- template for mathematical entities 73
- Temple, George 227

Index

- on specialization in mathematics 119
- term
 - use one and only one for a given entity 87
- tests
 - guidelines for taking 170
- textbook
 - absurd that professor is judge of its worth 26
 - author is not proposing doing away with traditional textbook 127
 - “A calculus textbook is a great big calculator.” 143
 - logical order of material in a 10
 - organization of material in a 6
 - traditional, author is not advocating doing away with 8
 - traditional, used as problem-solving device 67, 69
 - typical, is developed from lecture notes and ensures the continued need for a professor 6
 - typical, is the record of someone’s teaching of a subject, never of someone’s learning or using a subject 7
 - typical, logical organization of 7
 - what organization says about how book is to be used 6
 - why are there so many textbooks for each math subject? 27
- textbooks
 - explanation of "gestalt" type of textbook 29
 - why there are so many for a given subject 29
- theorems
 - look-up-ability of theorem statements 7
 - standard form for 108
- theorems and lemmas you are allowed to use in a proof 80
- "There is no royal road to learning." 127
- thinkers vs. orators 19
- throughput
 - definition of 51
- "To think is to classify." 152
- topology of problems 166
- transistors
 - placement and routing of 131
- Turing machine 55
- Turing Test for Environments 131
- Turing Test for intelligence of computers 131

Index

tutoring jr. high school student 170
typeset presentation of math
 tyranny 89
tyranny of exclusively typeset presentation of math 89

U

UCB -- see "University of California at Berkeley" 13
Ulam, S. M. 140
 on importance of pictures 60
 on thinking of mathematical ideas visually 95
understanding
 a definition of 143
"understanding-at-a-glance"
 examples of 204
"Universal Encyclopedia of Mathematics" 70
Universal Template for Mathematical Entities
 example of 204
University of California at Berkeley
 calculus lecture at 13
user and computer
 new view of 50
 traditional view of 50

V

vector calculus
 Environment for 69
 importance of drawings to illustrate concepts in 146
Viking Mars Lander 146

W

Wells, H. G. 23, 141
What = Where 150
"What good is this?" -- a perfectly legitimate question for students to ask of a math subject 182
What vs. How 45

Index

- in commuting to work 43
- in high school exam question 42
- in obtaining Master's degree 44
- in solving a quadratic equation 44
- table of examples 47
- examples of 42
- "Whatever can be made look-up-able, should be made look-up-able" ii, 138
- When All Else Fails... 163
- Whitehead, Alfred North
 - on importance of extending number of operations we can perform without thinking 134
 - university not needed merely to impart information 13
- "Why is this difficult?" 139
- Wolf, Fred Alan
 - on teaching the new physics 90
- woman at the keyboard in computer industry 170
- word-processor implementations of Environments 72
- writing style
 - unjustified high reputation of 104

Y

- "You can only learn math by doing it." 22
- "You guys just keep making this stuff up as you go along!" 157, 224
- "You have to learn everything in order to do anything" 64, 66, 124