A Possible Proof of Goldbach's Conjecture

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Statement of Conjecture

Goldbach's Conjecture, which was announced in 1742, asserts that each even positive integer greater than or equal to 4 is the sum of two prime integers. Thus, e.g., 12 = 5 + 7. The Conjecture is still unproved.

The Current Possible Proof, below, is followed by three versions of a Previous Possible Proof that we believe might be of interest to some readers. All are based on a strategy that has been effective with two other very difficult problems that we believe we have solved. The strategy begins with finding a structure that contains all possibilities and that shows crucial relationships between them.

In all cases except the last, the structure is the set of all sets of ordered pairs of odd positive integers, each pair in each set summing to the same even positive integer.

In the last case, the structure is the set of all odd primes.

Current Possible Proof

1. *Definition*: For all $k \geq 4$, the $2k$-set is the set of all ordered pairs of odd positive integers greater than 1 that sum to $2k$. Thus, e.g.,

the 2(4) = 8-set is the set {(3, 5), (5, 3)};

the 2(5) = 10-set is the set {(3, 7), (5, 5), (7, 3)};

the 2(6) = 12-set is the set {(3, 9), (5, 7), (7, 5), (9, 3)};

...

the $2(k) = 2k$-set is the set $\{(3, 2k - 3), (5, 2k - 5), ..., (2k - 5, 5), (2k - 3, 3)\}$);

...

2. Clearly:

(1) For each k, there is one and only one $2k$-set, whether or not Goldbach's Conjecture is true.

(2) Therefore, there is one and only one set of all $2k$ sets, whether or not Goldbach's Conjecture is true.

But (1) implies that if a $2k$-set lacks an ordered pair of primes, it is nevertheless the same $2k$-set that would exist if it contained an ordered pair of primes, which of course is absurd. And therefore Goldbach's Conjecture is true.

Preliminaries for Previous Possible Proofs

1. *Definition*: the *matrix* N is the infinite matrix whose rows and columns are labeled 3, 5, 7, 9, ... through all the odd positive integers.

Each cell of the matrix has coordinates (row, column). The contents of each cell is the sum row + column.

(For ease of understanding the following, the reader is encouraged to draw, and fill in, the matrix N for rows and columns 3, 5, 7, 9, 11, 13.)

2. *Definition*: The matrix N contains an infinite sequence of *diagonal*s. **For each $n \geq 1$, the nth diagonal is associated with the even positive integer $2k = 2n + 4$.** There are n cells in the nth diagonal. Each cell in the diagonal has, for its coordinates, a pair of odd positive integers that sum to $2k$.

We call each cell a "pair".

(The term "diagonal" is derived from the fact that all the elements of each diagonal lie on a diagonal line that is perpendicular to the main diagonal of the matrix N.)

3. For each $n \geq 1$, the nth diagonal is constructed as follows:.

We begin with the downward sequence $s(n)$ of left-hand elements in all pairs. That downward sequence is simply the sequence of n successive odd positive integers, starting with 3.

The first element is 3. *There is no possibility it can be something else*.

The next element is the next odd positive integer, namely 5. *There is no possibility it can be something else.*

The next element is the next odd positive integer, namely, 7. *There is no possibility it can be something else.*

…

[We proceed in this manner, through the successive odd positive integers up to and including the nth. The nth odd positive integer is $2n + 1$.]

The downward sequence $r(n)$ of right-hand elements in all pairs is just the reverse of $s(n)$, so *there is no possibility that any of these can be something else*.

.

(1)

Therefore there is one and only one possibility for each diagonal.

By way of an example, the following is the sixth diagonal. It is associated with $2k = 2(6) + 4 = 16$.

(3, 13)

(5, 11)
(7, 9)
(9, 7)
(11, 5)
(13, 3)

4. *Definitions*: A diagonal having no coordinates (p, q), where p, q are primes, we call a *counterexample diagonal*, because $2k$ is then an even positive integer that is not the sum of two primes, contradicting Goldbach's Conjecture.

A diagonal having at least one pair of coordinates (p, q), where p, q are primes, we call a *non-counterexample diagonal.*

Previous Possible Proof (First Version)

1. It is essential that the reader understand how a diagonal is constructed. This is described in step 3. The construction of a diagonal implies statement (1), which in turn implies that there is one and only one set of diagonals, regardless if counterexample diagonals exist or not.

2. But that implies that if the nth diagonal is a counterexample diagonal, it is also a non-counterexample diagonal, which is absurd.

Therefore counterexample diagonals do not exist, and Goldbach's Conjecture is true.

Previous Possible Proof (Second Version)

1. If one or more persons do not know if a given even positive integer $2k$ is associated with a counterexample diagonal or a non-counterexample diagonal, then for those persons there are two possibilities: (1) the diagonal is a counterexample diagonal, or (2) the diagonal is a non-counterexample diagonal.

(*Note*: The Goldbach Conjecture research community knows, by computer test, that at least for all j, $1 \leq j \leq m$, the jth even positive integer $2k$ is associated with a non-counterexample diagonal. The community knows that $m > 10^{18}$.)

2. Since, at the time of this writing, the Conjecture has not been proved or disproved, and given the *Note* in step 1, and the fact that there is an infinity of even positive integers $2k$ greater than m, there exists an infinity of $2k$ for which there are the two possibilities specified in step 1.

3. Steps 1 and 2 imply that there are two possibilities for an infinity of even positive integers $2k$, the smallest of these being greater than m in the *Note* in step 1.

4. However, in "Preliminaries for Previous Possible Proofs" on page 2, we showed that

(1) "*... there is one and only one possibility for each diagonal".*

Thus we have a contradiction that is brought about by the possibility that a diagonal can be a counterexample diagonal, and therefore Goldbach's Conjecture is true. (The contradiction is not brought about by the possibility that a diagonal can be a non-counterexample diagonal, because we know, by computer test, that it can.)

Previous Possible Proof (Third Version)

1, Let N' denote the infinite matrix whose columns are labeled with the consecutive odd primes, starting with 3, i.e.. 3, 5, 7, 11, 13, ...and similarly for rows.

The contents of the cell at location (row i, column j) is $i + j$. Thus the contents of the cell at location (7, 5) is 7 + 5 = 12.

The gist of our argument in this version of a Possible Proof is simply that there is one and only one matrix N' whether or not Goldbach's Conjecture is true, and therefore the set of even positive integers in the matrix is the same whether or not the Conjecture is true. This fact implies that a counterexample to the Conjecture does not exist..

And now to the details:

2. The matrix N' can be viewed as the result of an infinite succession of "squares". There is one square for each prime number. Thus, for example, the square for the prime 11 is

3	5	7	11
5	10	12	16
7	12	14	18
11	16	18	22

Each square is clearly a sub-matrix of N'.

For each square, there is a next successive square, which is obtained by attaching an "edge" to the right-hand side and to the lower side, of the previous square. Thus, in the case of the above example, the edge to be attached is

13
18
20
24
26

3. All the successive squares that contain all successive even positive integers up to some particular even integer $2k$, are the same whether or not the Conjecture is true. (We know from computer tests that $2k$ is greater than $4\ (10^{18})$.)

4. Let the square *S* be the first successive square such that all the even positive integers in *S* are not successive. It follows, from the way the successive squares are generated, that the even positive integers in all squares successive to *S*, will likewise not be successive.

5, But from the way that the successive squares are constructed, we can see that if there is such a square *S*, the resulting matrix *N'* must be the same as if there is no such square *S*.

(The skeptical reader is invited to tell us how the matrix *N'* that contains such an *S* could be different from the matrix *N'* that does not contain such an *S*.)

6. The implication is that the matrix *N'* is the same whether or not counterexamples to the Conjecture exist. Hence counterexamples do not exist, and the Conjecture is true.