

A Proof of Goldbach's Conjecture

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Statement of Conjecture

Goldbach's Conjecture, which was announced in 1742, asserts that each even positive integer greater than or equal to 4 is the sum of two prime integers. Thus, e.g., $12 = 5 + 7$. The Conjecture is still unproved.

First Proof

To prove the Conjecture, we must show that each even positive integer $2k$ is the sum of two odd primes, p, q . I.e., that $2k = p + q$.

We use proof by contradiction.

1. *Definition: diagonal for $2k$:* A diagonal for $2k$ is the set $\{(u, v) \mid u + v = 2k, \text{ where } u, v \text{ are odd positive integers } \geq 3\}$. We include (v, u) in the set.

Diagonals for $2k = 8$ through $2k = 22$ are shown in the following lists (see next page):

<u>$2k = 8$</u>	<u>$2k = 10$</u>	<u>$2k = 12$</u>
(3, 5)	(3, 7)	(3, 9)
(5, 3)	(5, 5)	(5, 7)
	(7, 3)	(7, 5)
		(9, 3)
<u>$2k = 14$</u>	<u>$2k = 16$</u>	
(3, 11)	(3, 13)	
(5, 9)	(5, 11)	
(7, 7)	(7, 9)	
(9, 5)	(9, 7)	
(11, 3)	(11, 5)	
	(13, 3)	
<u>$2k = 18$</u>	<u>$2k = 20$</u>	<u>$2k = 22$</u>
(3, 15)	(3, 17)	(3, 19)
(5, 13)	(5, 15)	(5, 17)
(7, 11)	(7, 13)	(7, 15)
(9, 9)	(9, 11)	(9, 13)
(11, 7)	(11, 9)	(11, 11)
(13, 5)	(13, 7)	(13, 9)
(15, 3)	(15, 5)	(15, 7)
	(17, 3)	(17, 5)
		(19, 3)

Fig. 1 Examples of Diagonals

Each ordered pair has a left-hand element and a right-hand element.

The set of all left-hand elements is called the *left-hand sequence*, and the set of all right-hand elements is called the *right-hand sequence*

The elements in the left-hand and right-hand sequences are fixed. The elements in a left-hand sequence are a sub-set of the elements of all left-hand sequences that follow in diagonals for larger $2k$ s, and similarly for the elements in a right-hand sequence.

2. How a diagonal for $2k + 2$ is constructed from a diagonal for $2k$:

(A) The left-hand sequence is extended to the next largest odd positive integer after the bottom element of the sequence. Thus, in the diagonal for $2k = 18$, the left-hand sequence is extended to 17.

This extended sequence now becomes the left-hand sequence of the diagonal for $2k + 2$.

(B) This new left-hand sequence for $2k + 2$ is now turned upside down and becomes the right-hand sequence in the diagonal for $2k + 2$.

3. *Definition*: a *counterexample diagonal*, or just a *counterexample* for short, is a diagonal in which there is no ordered pair (p, q) , where p, q are primes.

A *noncounterexample diagonal*, or just a *noncounterexample*, is a diagonal in which there is at least one pair (p, q) , where p, q are primes.

(At the time of this writing, each even positive integer $2k$, where $\{4 \leq 2k \leq (4)(10^{18})\}$, is known, by computer test, to be the sum of two primes, i.e., to be in conformity with Goldbach's Conjecture, and hence not a counterexample.)

4. From "How a diagonal for $2k + 2$ is constructed from a diagonal for $2k$ ", above, we claim the following:

Let d be any diagonal.

If d is a counterexample, then we denote d by d_c .

If d is a noncounterexample, then we denote d by d_n .

Then it follows from step 2 that $d_c = d_n$.

This is, of course, absurd, and therefore we conclude that there are no counterexamples, and hence Goldbach's Conjecture is true.

It is important that the reader understand the following distinction: suppose we have a very long sequence of results of flips of a fair coin. The sequence might begin 0, 1, 1, 0, 0, 0, 1, 0, 1, ...

For each $n \geq 1$, there is one and only one n th digit in the sequence. However that digit could be its "opposite" (where we are considering 1 and 0 to be "opposites").

That kind of thing cannot happen in the case of diagonals. No matter how big $2k$ is, we can describe exactly what the diagonal for $2k$ is. We cannot do the equivalent in the case of the sequence of 1s and 0s.

Second Proof

We show, as in "First Proof", that there is one and only one possibility for each diagonal, which implies (step 4 of "First Proof") that there are no counterexamples.

1. *Definition of the "number-slope"*:

A *number* is an odd, positive integer. A number can be a prime, like 5, or a composite, like 9.

A *number-slope* is the set of all occurrences of one number as the *right-hand* element in ordered pairs in an infinite succession of diagonals for $2k$. Thus, in the list of diagonals in Fig. 1, the 3-slope begins:

3 in (5, 3),

3 in (7, 3),
 3 in (9, 3),
 3 in (11, 3),
 3 in (13, 3),
 3 in (15, 3),
 3 in (17, 3),
 3 in (19, 3),

etc.

The reader can trace other number-slopes in Fig. 1 in “First Proof”.

(B) This new left-hand sequence for $2k + 2$ is now turned upside down and becomes the right-hand sequence in the diagonal for $2k + 2$.

The reason for the slope is that the extended left-hand sequence that results from the appending of the next odd positive integer below the left-hand sequence for the diagonal for $2k$, becomes, when turned upside-down, the right-hand sequence of the diagonal for $2k + 2$.

The appended odd positive integer becomes the first element in the right-hand sequence in the diagonal for $2k + 2$, and “pushes down” all the elements in what was the left-hand sequence for $2k$.

The number in a given number-slope is *fixed* in each diagonal. It cannot “disappear”, “be lost”, “move to another cell”, “change”, etc., in that diagonal. All of which is in keeping with the sentences in step 4 of “First Proof”:

2. Definition of the “number-horizontal line”:

A *number-horizontal line* is the set of all occurrences of one number as the *left-hand element* in ordered pairs in an infinite succession of diagonals for $2k$. Thus, in Fig. 1 in “First Proof”, the 7-horizontal line begins with the 7 in the ordered pairs

(7, 3), (7, 5), (7, 7), (7, 9), (7, 11), (7, 13), (7, 15), etc.

The reader can trace other number horizontal lines in Fig. 1.

The number in a given number-horizontal line is *fixed* in each diagonal. It cannot “disappear”, “be lost”, “move to another cell”, “change” in that diagonal, etc.

3. From steps 1 and 2 in this Proof we assert, as we did in “First Proof”:

“Let d be any diagonal.

If d is a counterexample, then we denote d by d_c .

If d is a noncounterexample, then we denote d by d_n .”

“Then it follows [from steps 1 and 2 in this Proof] that $d_c = d_n$.”

“This is, of course, absurd, and therefore we conclude that there are no counterexamples, and hence Goldbach’s Conjecture is true.”

Another way of expressing our conclusion is the following:

From step 3 in “First Proof” we have:

Definition: a *counterexample diagonal*, or just a *counterexample* for short, is a diagonal in which there is no ordered pair (p, q) , where p, q are primes.

From steps 1 and 2 in this Proof, we assert:

There is one and only one set of diagonals, whether or not a counterexample exists.

But that means there is no difference between a diagonal if it contains an ordered pair (p, q) , where p, q are primes, and that same diagonal if it does not contain such an ordered pair.

But that is absurd, “and therefore we conclude that there are no counterexamples, and hence Goldbach’s Conjecture is true.”