Appendix: Examples of Using an Environment to Solve a Problem

A Trigonometry Problem

This Appendix gives an example of using an Environment to solve math problems. In all cases, I have mentioned only the sections of the relevant Environment which were used in the course of solving the problem. In particular, I have not mentioned sections containing definitions of terms, although every term can be looked up in the complete Environment.

Problem Statement

Given the triangle shown in Fig. S-1, with FC = EC = 4, find BF = a and AE = b without solving a fourth degree equation.

Record of Process of Finding a Solution

Since we are trying to find out something about a triangle, I turned to the page in my Trigonometry Environment headed "triangle". The Universal Template in this case lists, among the operations we perform on triangles, "Solve for unknown sides and/or angles," and refers to a page headed, "triangle, how to solve for unknown sides and/or angles."

Step 1 of the heuristic is:

"1. Express all known sides in terms of known angles and sides using Pythagorean theorem and/or trigonometric identies."

This I did in several ways. I drew the line CD in case it might aid in arriving at other expressions.

The next step in the heuristic reads,

"2. Attempt to solve for unknown side or angle by setting up equations using trigonometric identities such as the law of cosines."

This I did, but each time wound up with a fourth degree equation. My intuition at this point was that the problem was really asking the potential solver to look closely at the fourth degree equation he came up with, and to notice a clever factoring that would easily yield the answer. So I turned to my Algebra Environment and looked up "equations, how to solve". The general formula for solving fourth degree equations was precisely what we are being asked not to use. The remaining factoring heuristics didn't seem to help.

I persisted in trying to find an easily factorable fourth degree equation, covering many sheets of paper in the process. Reluctantly, I came to the conclusion that either the cleverness required was beyond me, or else that my first intution was wrong.

I stared at Fig. S-1. Well, ABC was, after all, a right triangle. I turned to "triangle, right, theorems concerning", but found nothing that seemed of direct use. I stared at the figure again. Well, EDFC was clearly a square. I turned to the page headed "square, theorems concerning", but there were no theorems on squares embedded in triangles. More staring at Figure S-1. Well, clearly CD bisects angle ACB. I looked up "bisector, theorems concerning", and immediately noticed one that stated, "An angle bisector in a triangle divides the opposite side in the ratio of the sides adjacent to the angle bisected."

This meant I could write EQN 1 (below). It was immediately clear that one solution of the equations was that $a = b$, making angles BAC and ABC be 45° each. The solution shown then follows immediately.
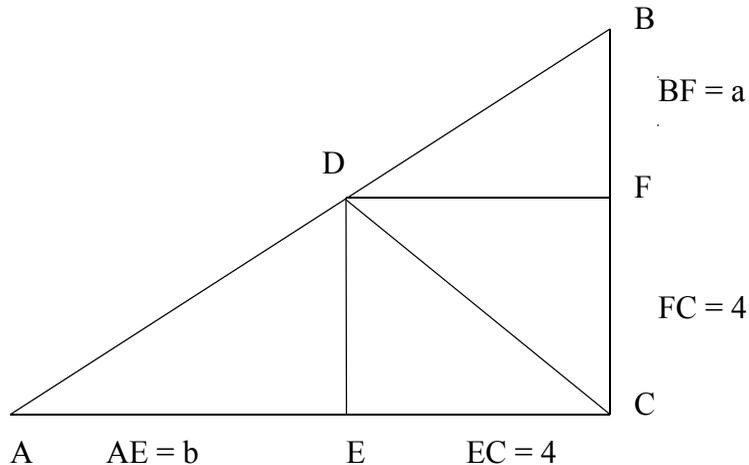
Figure S-1.

EQN 1

$$\frac{\sqrt{b^2 + 4^2}}{b + 4} = \frac{\sqrt{a^2 + 4^2}}{a + 4}$$

which implies $a = b$.

To find the hypotenuse, ADB, we can solve the equation

$$\sqrt{a^2 + 4^2} + \sqrt{b^2 + 4^2} = \sqrt{(a + 4)^2 + (b + 4)^2}$$

or, since $a = b$,

$$\sqrt{a^2 + 4^2} + \sqrt{a^2 + 4^2} = \sqrt{(a + 4)^2 + (a + 4)^2}$$

which implies,

$$2\sqrt{a^2 + 4^2} = \sqrt{2(a + 4)^2}$$

which implies, as the reader can verify,

$$a^2 - 8a + 16 = 0$$

and hence $a = 4$.
Thus, for the hypotenuse, ADB, we get

$$ADB = \sqrt{8^2 + 8^2} = 2^{7/2}$$